Vision Res. Vol. 24, No. 4, pp. 385–394, 1984
Printed in Great Britain.

0042-6989/84 $3.00 + 0.00

BINOCULAR CONTRAST SUMMATION—II. QUADRATIC SUMMATION

GORDON E. LEGGE

Department of Psychology, University of Minnesota, 75 East River Road, Minneapolis, MN 55455, U.S.A.

(*Received* 7 *July* 1982; *in revised form* 10 *August* 1983)

Abstract—Quadratic summation is presented as a rule that describes binocular contrast summation. The rule asserts that for left-eye and right-eye contrasts C_L and C_R, there is an effective binocular contrast C given by the formula:

$$C = \sqrt{(C_L)^2 + (C_R)^2}.$$

Pairs of left-eye and right-eye stimuli that produce equal values of C are equivalent. Quadratic summation is applied to the results of experiments in which stimuli presented to the two eyes differ only in contrast. It provides a good, first-order account of binocular summation in contrast detection, contrast discrimination, dichoptic masking, contrast matching and reaction time studies. A binocular energy-detector model is presented as a basis for quadratic summation.

Binocular vision   Binocular summation   Contrast

INTRODUCTION

This paper presents a theoretical description of binocular contrast summation. Most of the data come from studies using sine-wave gratings. The only cases to be considered will be those in which the two eyes are stimulated by sine waves of identical spatial frequency, orientation, and phase relative to fixation. Only the effects of *contrast* will be considered.

Quadratic summation will be presented as a description of binocular contrast summation. It gives a good, first-order account of a variety of phenomena, some of which have received no previous explanation.

Quadratic summation means that monocular signals add quadratically to form a binocular signal. For the case of contrast, the combination rule is stated as follows. For left-eye and right-eye contrasts of C_L and C_R, the binocular signal has *effective binocular contrast* C given by

$$C = \sqrt{(C_L)^2 + (C_R)^2}. \quad (1)$$

This formula assumes that the two monocular channels are equally sensitive. Equation (1) might be amended to account for some forms of ocular dominance by weighting C_L or C_R by some appropriate factor. In equation (1), a given value of the effective binocular contrast C might result from *monocular* stimulation of the left eye, *monocular* stimulation of the right eye, *binocular* stimulation in which equal contrasts are presented to the two eyes, or *dichoptic* stimulation in which unequal contrasts are presented to the two eyes. According to the quadratic summation rule, all such stimuli will have the same effect. Accordingly, equation (1) can be used to predict relationships among monocular, binocular and dichoptic stimuli.

Although equation (1) establishes contrast equivalence relations among monocular, binocular and dichoptic stimuli, it is not a model of contrast processing *per se*. For example, it does not predict the shape of the detection psychometric function, or the shape of the contrast discrimination function. On the other hand, if the form of such functions are known for monocular viewing, equation (1) predicts the corresponding form for binocular viewing, or vice versa.

In the following sections, the quadratic summation rule will be used to study contrast detection, contrast discrimination, dichoptic masking, contrast matching and reaction-time data. Within limits, quadratic summation provides a reasonable account of binocular summation phenomena associated with all of these.

CONTRAST DETECTION

The quadratic summation rule can be used to predict binocular thresholds from monocular thresholds. Suppose the monocular threshold contrast is C'_m. (In this paper, primed symbols refer to thresholds.) From equation (1), the effective binocular contrast, C, is equal to C'_m. Suppose that the threshold is obtained for a binocular grating in which equal contrasts are presented to the two eyes. Let the threshold contrast in this case be C'_b. From equation (1), the effective binocular contrast, C, associated with the binocular grating is just $\sqrt{2}C'_b$. Quadratic summation predicts that both the monocular and binocular thresholds will be determined by the same value of C. Therefore, monocular threshold C'_m and binocular threshold C'_b are related by the equation

$$C'_m = \sqrt{2}C'_b. \quad (2)$$

Quadratic summation predicts that the monocular threshold is $\sqrt{2}$ times greater than the binocular threshold. There is a great deal of evidence that monocular threshold contrasts are about $\sqrt{2} \approx 1.4$ times greater than monocular thresholds (see, e.g. Campbell and Green, 1965; Blake and Levinson, 1977). Legge (1984) found monocular/binocular threshold ratios of about 1.5, slightly greater than $\sqrt{2}$.

Contrast detection is characterized more completely by the psychometric function. Foley and Legge (1981) and Legge (1984) have shown that contrast detection can be represented by a relationship between detectability d' and contrast C of the form

$$d' = (C/C')^n.$$

C' is the contrast that corresponds to $d' = 1$, and may be taken as a definition of threshold contrast. n is an index of the steepness of the psychometric function, with typical values of 2 or slightly more (Foley and Legge, 1981; Legge, 1984). Although quadratic summation predicts that the monocular and binocular thresholds will differ by a factor of $\sqrt{2}$, it predicts that values of the steepness parameter n will be the same in the two cases. Legge (1984) measured monocular and binocular detection psychometric functions for 0.5-c/deg sine-wave gratings. No statistically significant differences between monocular and binocular steepness parameters were found. As a corollary, we may derive the relation between monocular detectability d'_M and binocular detectability d'_B. If $n = 2$, $d'_M = (C/C'_m)^2 = (C/\sqrt{2}C'_b)^2 = \frac{1}{2}(C/C'_b)^2 = \frac{1}{2}d'_B$. This means that the monocular detectability is equal to half the binocular detectability for a given contrast C. More generally, for contrasts C_L and C_R presented to the left and right eyes, the relation between binocular and monocular values of d' is given by

$$d'_B = d'_L + d'_R.$$

This relation is called *simple d′ summation* by Green and Swets (1974). It is a direct prediction of quadratic summation. In the contrast-detection data of Legge (1984), there was a tendency for binocular detectabilities to exceed the sum of the monocular detectabilities, but the tendency was not statistically significant.

Anderson and Movshon (1981) have measured contrast thresholds for dichoptic stimuli with monocular components having unequal contrasts. What does quadratic summation product in this case? Since a fixed level of performance should correspond to a fixed value of C in equation (1), thresholds for unequal component contrasts should obey the relation

$$(C_L)^2 + (C_R)^2 = \text{constant}.$$

The data of Anderson and Movshon are consistent with this prediction.

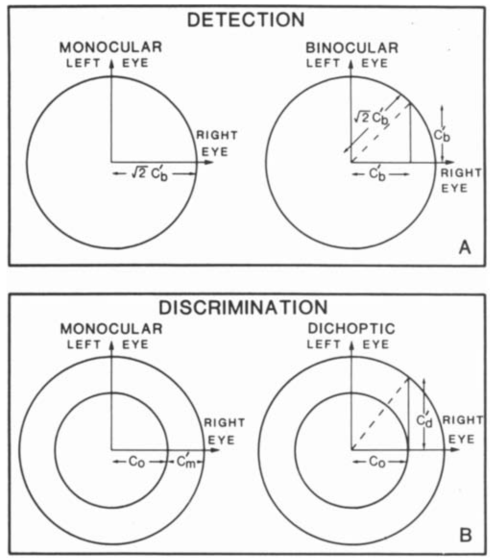

Fig. 1. Geometrical model of quadratic summation. In the drawing, distance represents effective binocular contrast. Horizontal and vertical distances represent right-eye and left-eye contrasts respectively. (A) Monocular and binocular detection. (B) Monocular and dichoptic discrimination.

Figure 1(A) provides a pictorial representation of monocular and binocular detection, according to quadratic summation. In the diagram, the "effective binocular contrast" corresponds to distance from the center of the circle. Horizontal distance corresponds to right-eye contrast, and vertical distance to left-eye contrast. A criterion value of C, associated with threshold, is represented by a circle. The criterion value can be achieved by a monocular contrast of $\sqrt{2}C'_b$, or by equal monocular contrasts C'_b added quadratically, that is, at right angles.

CONTRAST DISCRIMINATION

In contrast discrimination experiments, observers are typically required to discriminate between two sine-wave gratings that differ only in their contrasts, C and $C + \Delta C$. The smallest value of ΔC that allows for reliable discrimination is the *contrast increment threshold*. The relationship between increment threshold ΔC and background contrast C is sometimes called the *contrast discrimination function*. When the background contrast is 0, contrast discrimination reduces to contrast detection.

The contrast discrimination function for a given stimulus can be measured monocularly or binocularly. What relationship between the two does quadratic summation predict? Suppose the background contrast is C_0. According to equation (1), the corresponding effective binocular contrast C is equal to C_0 for monocular viewing, and $\sqrt{2}C_0$ for binocular viewing. Suppose an increment ΔC is added to the background. The increment of the effective binocular contrast is just ΔC for monocular viewing, and $\sqrt{2}\Delta C$ for binocular viewing. In a plot of increment threshold ΔC vs background contrast C, quadratic summation predicts that the monocular results can be derived from the binocular results by multiplying the binocular thresholds by $\sqrt{2}$ and plotting them at background contrasts that are increased by a factor of $\sqrt{2}$. Graphically, this amounts to shifting the binocular data vertically by a factor of $\sqrt{2}$, and rightward by a factor of $\sqrt{2}$.

This prediction was examined experimentally. Monocular and binocular contrast discrimination functions were measured for 0.5-c/deg sine-wave gratings. The stimuli and apparatus were described in detail by Legge (1984). The forced-choice paradigm has been described in detail by Legge and Kersten (1983). In short, observers viewed a 340 cd/m^2 CRT display. A vertical septum divided the screen into two halves, one for viewing by each eye. Fixation marks, base-out prisms and spectacle lenses ensured a fused image. A computer generated digital waveforms that were applied to the Z-axis of the CRT display so that gratings could be presented to either or both sides of the screen. Threshold estimates were obtained from forced-choice staircases with six reversals (Wetherill and Levitt, 1965).

Two observers participated in the experiments. Both were well practiced. Neither observer had significant eye differences in detection thresholds for 0.5 c/deg. For each of seven background contrasts, six binocular and monocular (right eye) threshold estimates were obtained for observer K.J. For G.D., four such estimates were obtained. During monocular stimulation, the contralateral eye continued to view a uniform field, apart from fixation marks, of the same mean luminance.

The two panels of Fig. 2 show monocular and binocular contrast discrimination functions for the two observers. The points are geometric means of the several threshold estimates. The bars represent $\pm$ 1 SE. The four discrimination functions have the familiar "dipper shape" (Legge and Foley, 1980; Legge and Kersten, 1983). First, consider the binocular data (open circles). For background contrasts of 2% and above, the data have been fitted by straight lines. The solid curves through the remainder of the data have been fitted by eye. The slopes of the straight line portions are 0.54 and 0.61 for K.J. and G.D. respectively. Accordingly, for both observers, suprathreshold binocular contrast discrimination can be described by a power law relation between increment contrast and background contrast, with an exponent near 0.6. These results are in agreement with similar findings for sine-wave gratings (Legge and Foley, 1980; Legge, 1981), light and dark bars (Legge and Kersten, 1983) and difference-of-Gaussians (Wilson, 1980).

Given the results for binocular contrast discrimination, the quadratic summation rule predicts the form of the monocular contrast discrimination function. It is found simply by shifting the binocular curve upward and to the right by factors of $\sqrt{2}$. The dashed lines in Fig. 2(A) and 2(B) constitute this prediction. The triangles represent the monocular data. The monocular results are in reasonable agreement with the prediction. In particular, for low background contrasts, and for contrast detection, the monocular/binocular threshold ratio is greater than at high contrast. In fact, both the data and predictions agree that for suprathreshold background contrasts, there is very little difference between monocular and binocular thresholds. In other words, there is very little binocular advantage in suprathreshold contrast discrimination. The same conclu-

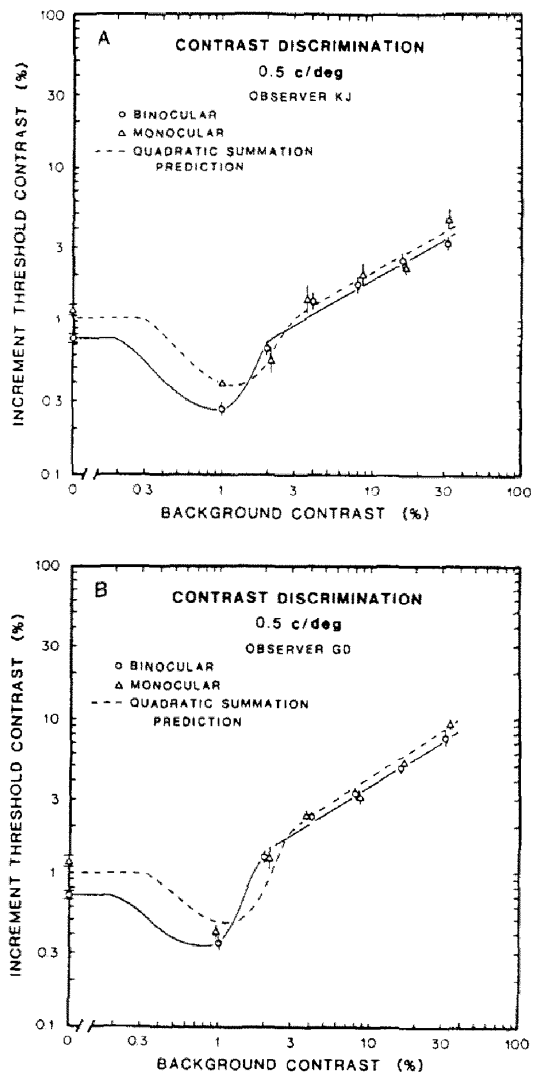

Fig. 2. Monocular and binocular contrast discrimination functions. Contrast increment thresholds are plotted as a function of background contrasts for 0.5-c/deg sine-wave gratings. Each point is the geometric mean of several threshold estimates, each derived from a forced-choice staircase. Bars represent $\pm$ 1 SE. Solid curves have been drawn to fit the binocular data. The dashed curves are the quadratic summation predictions for monocular contrast discrimination. The monocular predictions are derived from the solid curves by vertical and horizontal shifts of factors of $\sqrt{2}$. (A) Observer K.J. (B) Observer G.D.

sion was reached previously by Legge (1984). He measured psychometric functions for contrast discrimination for background contrasts of 5 and 25%. Much less binocular summation was observed in these cases than for contrast detection.

DICHOPTIC MASKING

Binocular contrast interactions have been revealed by contrast masking studies (Legge, 1979; Levi *et al.*, 1979). Two cases have been compared. In *monocular masking*, threshold contrasts for test gratings presented to one eye were measured when masking gratings were presented to the same eye. In *dichoptic masking*, thresholds for test gratings presented to one eye were measured when masking gratings were presented to the contralateral eye. In both cases, masking was found to be spatial-frequency and orientation specific. However, Legge (1979) observed a very puzzling difference between monocular and dichoptic masking. When masker and test were identical, except for contrast, dichoptic masking produced much greater threshold elevation than monocular masking. No quantitative explanation has been offered for this difference.

In Fig. 3(A) and 3(B), thresholds obtained in monocular and dichoptic masking experiments have been replotted from Legge (1979). The data pertain to conditions in which test and maskers differed only in contrast, and may be referred to as monocular and dichoptic discrimination. Data from several spatial frequencies are plotted in normalized coordinates in which all contrasts have been divided by the detection threshold contrast. As a result, normalized contrasts of 1.0 correspond to threshold contrast. Except for the 0.5-c/deg data (see below), each point is the geometric mean of 12 threshold estimates, each from a two-alternative forced-choice staircase, pooled across two observers. Each forced-choice trial consisted of two 200-msec intervals. The "masker" or "background" was presented in both intervals, and the test was added in one. From the observer's point of view, both the monocular and dichoptic tasks involved a discrimination in which they sought to identify the interval having the higher apparent contrast. In Fig. 3(A) and 3(B), solid curves have been drawn through the data. Best-fitting straight lines have been fit to the data at medium and high contrasts, and smooth curves drawn through the low-contrast data.

In Fig. 3, data at 0.25, 1, 4, and 16 c/deg come from Legge (1979), but the 0.5-c/deg data have been added as a replication. The 0.5-c/deg thresholds were obtained from psychometric functions for monocular and dichoptic contrast discrimination collected with the methods described by Legge (1984). Data are for one observer, D.P., and are representative of data collected from three observers. Each of the 0.5-c/deg points in Fig. 3 is a geometric mean of 8 threshold estimates (four right eye and four left eye), each derived from a psychometric function. The dichoptic discrimination trials were interleaved with the monocular discrimination trials.

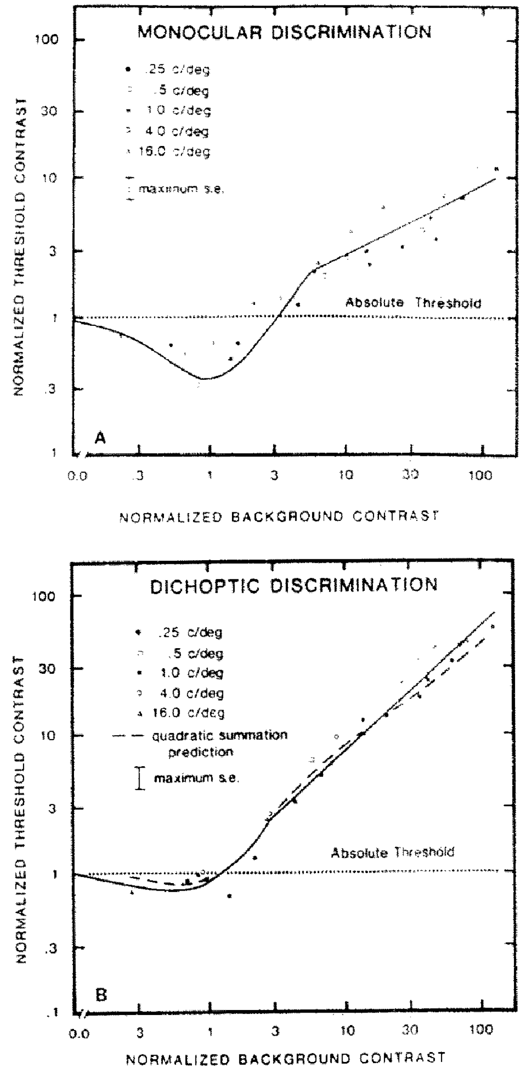

Fig. 3. Monocular and dichoptic contrast discrimination. Test thresholds are plotted as a function of background contrast. Test and background were identical in spatial frequency, orientation, and phase. Contrasts have been normalized by the appropriate contrast detection threshold. Data for 0.25, 1, 4 and 16 c/deg have been replotted from Legge (1979). Data of 0.5 c/deg were obtained with the methods described by Legge (1984). Bars show maximum standard error ± 1 SE. (A) Monocular discrimination: background gratings and test increments were presented to the same eye, while the contralateral eye viewed a uniform field of the same mean luminance. The straight-line portion of the solid curve through the data has a slope of 0.5 in the log–log coordinates. A smooth curve has been drawn by eye through the low-contrast data. (B) Dichoptic discrimination: the background grating was presented to one eye while the test grating was presented to the other. The solid curve through the data is the fit provided by Legge (1979). The straight-line portion of the curve has a slope of 0.9 in the log–log coordinates. The dashed line through the data is the prediction of quadratic summation. It was derived from the solid curve through the monocular data in panel (A), using equation (4).

In the normalized units of Fig. 3, data at the five spatial frequencies appear to follow the same functions. For monocular discrimination, we have the familiar dipper, characterized by "facilitation" at low background contrasts, and a steady rise at high background contrasts. By comparison, the dichoptic discrimination data exhibit a weaker facilitation effect at low contrasts, and a much steeper rise at high background contrasts.

The quadratic summation rule can be used to predict the dichoptic discrimination results from the monocular results. Suppose the background contrast presented to one eye is C_0. From equation (1), the corresponding effective binocular contrast C is also C_0. Suppose the monocular increment threshold is C'_m. This means that the observer can just discriminate a grating having contrast $C_0 + C'_m$ from a grating having contrast C_0, if the two are presented monocularly. The effective binocular contrast corresponding to the higher contrast grating is just $C_0 + C'_m$. Now, suppose that instead of adding the increment to the background in the same eye, a contralateral test grating is presented. Let the threshold for this dichoptically presented test grating be C'_d. From equation (1), the effective binocular contrast C associated with a grating of contrast C_0 presented to one eye and a grating of contrast C'_d to the other is $\sqrt{(C_0)^2 + (C'_d)^2}$. Quadratic summation predicts that both the monocular and dichoptic thresholds will be determined by the same value of C. Therefore, monocular threshold C'_m and dichoptic threshold C'_d for a given background contrast C_0 are related by the equation

$$C_0 + C'_m = \sqrt{(C_0)^2 + (C'_d)^2}. \quad (3)$$

Algebraic manipulation of equation (3) gives C'_d as a function of C_0 and C'_m

$$C'_d = \sqrt{(C_0 + C'_m)^2 - (C_0)^2}. \quad (4)$$

Equation (4) is the quadratic summation prediction for the dichoptic threshold C'_d, given the monocular threshold C'_m for background contrast C_0.

Equation (4) was used in conjunction with the monocular discrimination results of Fig. 3(A) to predict dichoptic discrimination. Values along the solid curve in Fig. 3(A) were "plugged" into equation (4) to generate the dashed curve in Fig. 3(B). This dashed curve is the quadratic summation prediction for dichoptic discrimination. The dashed curve lies very close to the solid curve through the data, and provides a good account of the results. In agreement with the results in Fig. 3(B) and with measurements of Blake and Levinson (1977), quadratic summation predicts a reduced facilitation effect for low-contrast backgrounds in dichoptic discrimination. Also in agreement with the data, quadratic summation predicts a steeper rise in threshold contrast for dichoptic compared with monocular suprathreshold backgrounds. The success of the quadratic summation rule in predicting the unusual dichoptic discrimination results, without any free parameters, is perhaps its major accomplishment.

The reason why quadratic summation predicts higher thresholds in the dichoptic case can be seen in Fig. 1(B). As in Fig. 1(A), distance from the center of the circles corresponds to effective binocular contrast. Contrast discrimination amounts to increasing the effective binocular contrast by some criterion amount. The just-discriminable pair of effective binocular contrasts are represented in the diagram by concentric circles. For a background contrast of C_0 presented to one eye, an increment may be added in the same eye or the other eye. In the latter case, the addition is at right angles. By comparing the monocular and dichoptic cases in Fig. 1(B), it is clear that the contrast added at right angles (quadratic summation) must be considerably greater than the contrast added linearly in order to reach the outer circle. Figure 1(B) makes it easy to verify equation (4) as well. The vertical line of length C'_d is one side of a right triangle. The side adjacent at the right angle has length C_0 and the hypotenuse has length $C_0 + C'_m$. Equation (4) immediately follows from the Pythagorean theorem.

CONTRAST MATCHING AND REACTION TIME STUDIES

Psychophysical paradigms other than those relying on threshold measurements can be used to assess binocular summation.

Legge and Rubin (1981) performed a binocular contrast matching experiment, similar to Levelt's (1965) binocular brightness matching experiment. They used a matching procedure to find pairs of unequal monocular contrasts of sine-wave gratings whose binocular appearance matched a standard stimulus. The standard consisted of equal-contrast gratings presented to the two eyes. They found that their data could be fit by functions having the form

$$(C_L)^n + (C_R)^n = \text{constant} \quad (5)$$

where C_L and C_R are the left-eye and right-eye contrasts that combine to match a particular standard. Quadratic summation predicts such a relation with $n = 2$. Legge and Rubin (1981) found values of n ranging from 1.6 to 4.3, but with most values clustering near 2. Values of n tended to be slightly higher for higher standard contrasts than for lower ones. Their results were similar at 1 and 8 c/deg. As a first approximation, the binocular contrast matching results are described by quadratic summation. Birch (1979) and Iverson, Movshon and Arditi (1981) have conducted similar measurements of binocular contrast matching. Their results generally conform to quadratic summation as well.

There are two experiments in which reaction times have been measured as a function of contrast for gratings viewed binocularly and monocularly (Harwerth *et al.* 1980; Blake *et al.* 1980). If reaction times are based on some property of the "binocular signal" that results from combination of the monocular inputs, the quadratic summation model predicts that identical reaction times will occur for monocular gratings having contrasts $\sqrt{2}$ times greater than binocular gratings. In both studies, monocular/binocular contrast ratios near $\sqrt{2}$ for criterion reaction times were found for near-threshold stimuli. Actually, Harwerth *et al.* found values ranging from 1.44 to 1.74, slightly greater than $\sqrt{2}$. However, for suprathreshold contrasts, Blake *et al.* (1980) found contrast ratios that increased to values near 2. On the other hand, Harwerth *et al.* (1980) found substantial individual variation in the monocular/binocular contrast ratios for suprathreshold gratings with some values exceeding $\sqrt{2}$ and others being less. Apparently quadratic summation provides a reasonable account of the near-threshold reaction time results, but cannot account for the variable suprathreshold findings.

Magnitude estimation experiments would be another way of testing quadratic summation. It is known that perceived contrast can be described as a threshold-corrected power function of stimulus contrast for sine-wave gratings. The exponent appears to lie somewhere in the range from 0.7 (Gottesman *et al.*, 1981) to 1.0 (Cannon, 1979). For suprathreshold stimuli, quadratic summation predicts that both monocular and binocular functions should have the same exponent, but should differ by a scale factor in overall magnitude. The scale factor should be $(\sqrt{2})^n$ where n is the exponent of the power function. For n in the range 0.7–1.0, quadratic summation predicts that binocular magnitude estimates should be 27–41% greater than monocular estimates for the same stimulus contrast. This experiment has not yet been done. Stevens (1967) did a comparable experiment in which he compared monocular and binocular brightness estimates. The brightness power function has an exponent of about 0.33, so quadratic summation would predict a scale factor of $(\sqrt{2})^{0.33} = 1.12$ for this case. This is exactly what Stevens found, a slight difference between monocular and binocular judgments with a mean difference of about 1 dB.

QUADRATIC SUMMATION AND THE BINOCULAR ENERGY DETECTOR

A rule of binocular contrast summation must specify which combinations of left-eye and right-eye contrasts are equivalent stimuli. Quadratic summation is such a rule. In this section, we address two major questions. What sort of model of binocular interaction might yield quadratic summation? Can this form of binocular contrast interaction be related to existing models of contrast coding in vision?

The quadratic summation rule contains terms in squared contrast, suggestive of a square-law device. Such a device is at the heart of the energy-detector model of signal-detection theory (Green and Swets, 1974, Chap. 8). The energy detector has been a valuable heuristic for studies of auditory psychophysics. A simple extension of the energy-detector model to the case of binocular contrast summation manifests quadratic summation and at the same time closely resembles some current models of contrast coding. Figure 4 presents a block diagram of the *binocular energy-detector* model. Taken separately, each monocular channel is equivalent to the energy detector described by Green and Swets.

Suppose the stimuli are sine-wave gratings. Their one-dimensional luminance profiles are

$$L(x) = L_0[1 + C\sin(2\pi f x)]$$

where L_0 is the mean luminance, C the contrast, f the

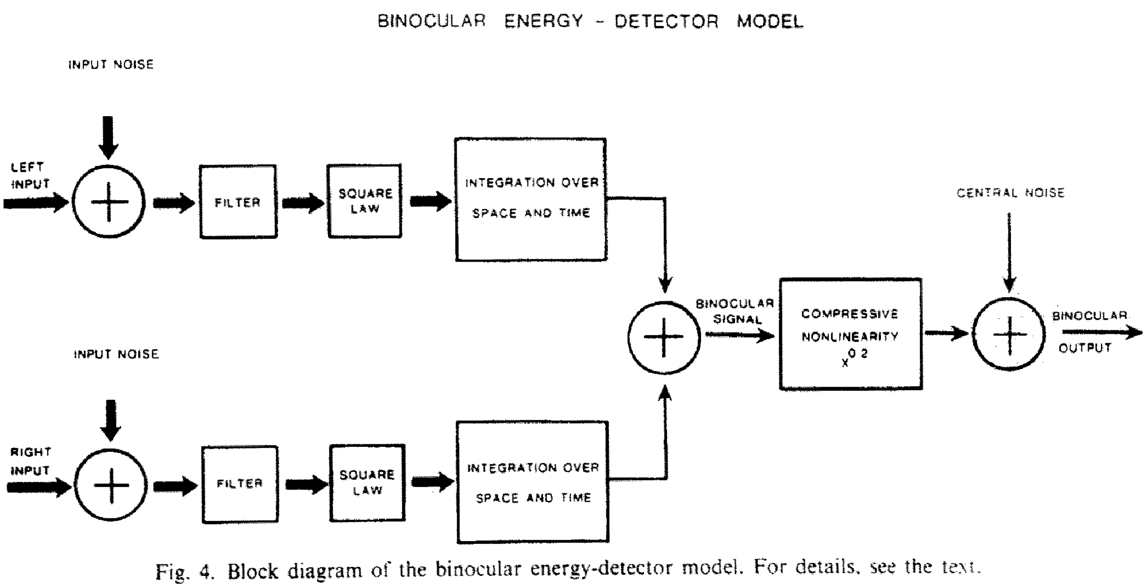

Fig. 4. Block diagram of the binocular energy-detector model. For details, see the text.

spatial frequency, and x the position. The corresponding contrast function $C(x)$ is defined to be (Linfoot, 1964)

$$C(x) = [L(x) - L_0]/L_0 = C \sin(2\pi f x).$$

More generally, the contrast function depends on two spatial dimensions as well as time, and is written $C(x,y,t)$. We take the contrast functions associated with the left and right stimuli as the inputs to the model in Fig. 4. The broad arrows indicate transmission of an entire function of space and time, such as a contrast function. The thin arrows represent transmission of a single number per stimulus presentation (see below). Zero-mean, constant-variance Gaussian noise is added to the input. This noise limits performance near threshold. Pelli (1981) has provided psychophysical evidence for the existence of such noise. The noise-perturbed contrast function is passed through a linear spatiotemporal filter. (This filter may be constructed from a set of identical receptive fields distributed over space. The outputs of the receptive fields constitute a discrete approximation to the convolution of the input with the weighting function of the receptive fields.) The output, $H(x,y,t)$, of the filter is then squared. The output, $H^2(x,y,t)$, of the square-law device is then integrated over the spatial and temporal extent of the stimulus to yield a single number for each stimulus presentation. For a given stimulus, the output of the integrator is a random variable that is approximately Gaussian.* The pair of noisy outputs from the two monocular channels are added to form the *binocular signal*. The binocular signal is also approximately Gaussian.

The remaining elements of the model are required to make it consistent with contrast-discrimination data. Suprathreshold contrast discrimination usually obeys a power-law relation between incremental threshold ΔC and background contrast C with an exponent near 0.6 (Legge, 1981). The growth of ΔC can be accounted for by a compressive transformation of the binocular signal and the addition of noise. In Fig. 4, the binocular signal is subjected to a compressive power-law transformation with exponent 0.2, followed by the addition of zero-mean, constant-variance noise. This noise is termed *central noise* to distinguish it from the input noise. (Over a two log-unit range of contrasts, a 0.2-power law can be approximated by a log transformation. A log transformation could have been used in Fig. 4.)

At low contrasts, the input noise dominates and limits performance. As contrast rises, the variance contributed by the central noise to the binocular output remains constant, but the variance contributed by the input noise is attenuated by the compressive nonlinearity. Eventually, the central noise dominates,† and ΔC grows with C.

The combination of a 0.2-power law with constant-variance central noise is equivalent to adding a signal-dependent noise with standard deviation proportional to the 0.8-power of the binocular signal.‡ Such an alternate formulation could have been used in Fig. 4. In fact, there exists electrophysiological evidence for signal-dependent noise. Tolhurst *et al.* (1981) measured the mean and standard deviation of the number of spikes elicited by passage of one cycle of a drifting grating through the receptive fields of 20 cat simple and complex cells. Over a range of contrasts between threshold and saturation, the standard deviation increased as a power function of the mean with exponent in the range 0.5–0.7. Green and Swets (1974) used signal-dependent noise at the output of the energy detector to model Weber's law for pure-tone intensity discrimination. The compressive nonlinearity is shown in Fig. 4 rather than signal-dependent noise for easier comparison with existing models of contrast discrimination (see below).

Finally, the observer's decision in a psychophysical task is based upon values of the binocular output. For example, in a temporal, two-alternative forced-choice trial, the observer chooses the interval in which the value of the binocular output is greatest.

Some of the properties of this model that are relevant to binocular summation and contrast discrimination are summarized below. Proofs are not given, but the computations closely follow those presented by Green and Swets (1974, Chap. 8).

Quadratic summation

For a given stimulus, the mean value of the integrator's output is equal to a constant (whose value depends on stimulus size, filter bandwidth and noise

---

*The integrator's output is actually the sum of N χ^2 variables, where N is determined by the spatial and temporal extent of the stimulus and the filter's bandwidth. Unless N is small, the sum is approximately Gaussian, by the central limit theorem. For stimuli that are narrowly confined in space and time, the approximation breaks down.

†To see this, consider the incremental gain which is equal to the derivative of the binocular output with respect to contrast. Denote the output by S. S rises as the $2 \times 0.2 = 0.4$ power of contrast. Therefore, the incremental gain is proportional to $d/dC(C^{0.4})$ which is proportional to $C^{-0.6}$. Therefore, the contribution of the input noise to the binocular output has standard deviation that drops as the -0.6 power of contrast and variance that drops as the -1.2 power. As contrast rises, the input noise rapidly becomes insignificant compared with the constant-variance central noise.

‡Discrimination performance is determined by the ratio of mean to standard deviation of the decision variable. In this case, the decision variable is the binocular output S. At high contrasts, the mean of S rises as $C^{0.4}$ and its standard deviation is independent of contrast. Therefore, the ratio of mean to standard deviation rises as the 0.4-power of contrast. For the case of signal-dependent noise, the mean rises as squared contrast, and the standard deviation rises as the 0.8-power of the mean or 1.6-power of contrast. Therefore, the ratio of mean to standard deviation also rises as the 0.4-power of contrast.

spectral density, and is significant only near threshold) plus a term proportional to squared contrast. Accordingly, the mean value of the binocular signal is equal to a constant plus a sum of terms that are proportional to left and right squared contrast. This is suggestive of the basis for the quadratic summation behavior of the binocular energy detector. It is important to note that this behavior depends crucially on the existence of the square-law nonlinearity *prior* to the point of binocular combination.

Computations show that this version of the energy detector manifests all the forms of quadratic summation described in earlier sections for detection and discrimination—the $\sqrt{2}$ relation between monocular and binocular thresholds, parallel psychometric functions for monocular and binocular detection, simple d' summation, parallel and nearly overlapping monocular and binocular discrimination functions at high contrast, and the marked differences between dichoptic and monocular discrimination functions. If contrast magnitude judgments depend on the mean value of the binocular output, the model predicts the quadratic form of suprathreshold binocular contrast matches and the relation between monocular and binocular contrast magnitude estimates.

Contrast coding

Several models have been proposed recently to account for the shape of contrast-discrimination functions (Legge and Foley, 1980; Carlson and Cohen, 1978; Wilson, 1980; Burton, 1981). These models all have some form of nonlinear relation between internal response and stimulus contrast, as well as one or more sources of internal noise. They bear a close resemblance to the energy-detector model.

According to the Legge and Foley model, the input is first passed through a linear spatial-frequency filter, identical in conception to the band-pass filter of the energy detector. The filter's output is subjected to a nonlinear transformation. The input/output relation of the nonlinearity is positively accelerated at low contrasts and compressive at high contrasts. The accelerating portion of the nonlinearity is described by a power law with exponent 2.4, quite close to the value of 2 expected from a square-law device. The compressive portion of the nonlinearity is described by a power law with exponent 0.4. The model presented in Fig. 4 has two power-law transformations in tandem with an overall exponent of $2 \times 0.2 = 0.4$. In this respect, the high-contrast behavior of the energy-detector model in Fig. 4 is identical to Legge and Foley's nonlinear transducer model. The advantage of the energy-detector model in the present context is that it permits the binocular combination to take place *after* the squaring but *before* the compressive transformation that limits suprathreshold discrimination performance. This means that the model accounts for both quadratic summation phenomena of binocular interaction *and* characteristics of contrast discrimination. Finally, it should be noted that the integrator in Fig. 4 has as its counterpart a form of spatial summation in the Legge and Foley model.

In short, the recent models of contrast discrimination, and in particular the Legge and Foley model, are highly similar to the energy-detector model of Fig. 4. Both types of models account for the accelerated form of the contrast-detection psychometric function, the linearity of the increment-detection psychometric function, the dipper shape of the contrast-discrimination function, and the 0.6-power law of suprathreshold contrast discrimination.

It may be concluded that the model diagrammed in Fig. 4 represents a synthesis of models of binocular contrast summation and contrast coding that provides a first-order account of a diverse set of contrast phenomena.

The energy-detector model of this section should be distinguished from a model based on luminous-energy summation. According to the latter, luminous energy that is presented to corresponding regions of the two retinas is summed linearly. Therefore, a given quantity of luminous energy at monocular threshold can be divided equally between the two eyes and remain at binocular threshold. The most direct translation of this model to the domain of contrast would require that the two eyes linearly sum contrasts. This is certainly inconsistent with the evidence for quadratic summation. However, if we define *contrast energy* to be the integral over space and time of the squared contrast function, then the binocular energy detector of Fig. 4 *does* sum contrast energies from the two eyes. In fact, the term "energy detector" is used because of the computation of an energy-like quantity in the model.

The energy-detector model has been used widely in auditory psychophysics, but less often in vision. Rashbass (1970) used a variant of the model to account for the detection of brief luminance changes separated by different durations. His model included a linear, band-pass, temporal filter, a square-law device, integration over time, and a threshold device, but no explicit sources of noise. The model was not applied to spatial contrast, binocular interation or discrimination.

As Rashbass (1970) pointed out, there is an absence of quantitative physiological evidence for neurons with square-law input/output functions. However, a square-law might be synthesized in a number of ways from an ensemble of neural responses. As an illustration, suppose that a given set of neurons exhibits a linear relation between response R (spike rate) and contrast C. Let the neurons have staggered thresholds so that they operate over different ranges of contrast. Assume that the number N of active neurons is proportional to the contrast level C. Then, the overall response, summed across all active neurons, is equal to NR and is proportional to C^2.

RELATION TO OTHER MODELS OF BINOCULAR SUMMATION

Quadratic summation is a rule that describes binocular contrast summation. It is a special case of vector summation. In the geometrical model of Fig. 1, quadratic summation is represented by an angle of 90° between the left-eye and right-eye components. If vectors of length C_L and C_R are added at some arbitrary angle a, the length C of the summation vector is given by

$$C^2 = (C_L)^2 + (C_R)^2 + 2C_L C_R \cos(a).$$

When $a = 0$, we have linear summation and $C = C_L + C_R$. When $a = 90°$, we have quadratic summation, and $C^2 = (C_L)^2 + (C_R)^2$. Curtis and Rule (1978) were able to fit binocular brightness magnitude estimates with a vector summation of monocular brightnesses. Their data required an angle a equal to 113°. It is possible that some angle different from 90° would provide an overall better fit to the variety of data discussed in this paper. If so, binocular-contrast summation could be described as vector summation with the specified value of angle a. However, simplicity of conception and calculation argue strongly for quadratic summation as a starting point.

Many models have been used to account for phenomena of binocular summation. *Probability summation* and the *integration model* described by Green and Swets (1974, Chap. 9) are among the most common. Neither of these models adequately describes monocular and binocular contrast-detection data. For a detailed discussion, see Legge (1984).

Squared terms often appear in weighted-summation models of binocular brightness combination. For example, according to Engel (1967, 1969), binocular brightness B_B is a weighted sum of monocular brightnesses B_L and B_R

$$(B_B)^2 = (W_L B_L)^2 + (W_R B_R)^2.$$

The weighting coefficients W_L and W_R are related to the integral of a squared autocorrelation function computed across space upon some function of brightness. The model appears to account for some binocular brightness phenomena, but, as pointed out by Blake and Fox (1973), is hard to distinguish from the much simpler luminance-averaging model of Levelt (1965). The weighted-summation models of binocular brightness are not immediately applicable to detection or discrimination data. Moreover, Legge and Rubin (1981) concluded that weighted-summation models do not give an adequate description of their binocular contrast matching results.

Campbell and Green (1965) developed the first threshold model of binocular-contrast summation. According to their model, monocular signals are added linearly to form a binocular signal. The monocular signals are perturbed by independent sources of Gaussian noise. The addition results in a binocular signal-to-noise ratio that is $\sqrt{2}$ times greater than the monocular signal-to-noise ratio. This $\sqrt{2}$ factor accounts for the difference between monocular and binocular contrast thresholds. Implicit in the model is a linear relation between d' and contrast. Such a relation is inconsistent with the accelerating psychometric functions measured by Legge (1984). Moreover, the model has not been developed to deal with discrimination data. The binocular energy-detector model of Fig. 4 is really an elaboration of the Campbell and Green model that takes these limitations into account.

The treatment of binocular summation given in this paper is limited to cases in which the monocular stimuli differ only in contrast. A more complete treatment would take into account differences along several stimulus dimensions, including spatial frequency, orientation and disparity. Since there is ample evidence for visual selectivity along all of these dimensions, it is likely that such a treatment would involve channel theory. Consider, for example, *disparity*. When identical sine-wave gratings are presented to the two eyes but with unequal phase relative to the fixation points, nonzero disparity is introduced. The observer perceives a sine-wave grating that lies in depth relative to the plane of fixation. Psychophysical evidence for disparity selectivity comes from adaptation studies (Blakemore and Hague, 1972; Felton *et al.*, 1972), and noise-masking studies (Rubin, 1983). The binocular energy-detector model might be extended to account for disparity selectivity by assuming the existence of two such detectors working in parallel. One of the detectors would be tuned to zero disparity. The second would be tuned to a disparity corresponding to a 180° relative phase shift but otherwise would operate like the first. For a given stimulus, the relative activity of the two detectors would convey disparity information. Since a 180° phase-shift of a sine wave is equivalent to a sign reversal, the two detectors would be tuned to sums and differences of binocular combinations of sine-wave stimuli.* This scheme is qualitatively similar to the two-channel model proposed by Cohn and Lasley (1976). Their model was developed to describe threshold data for binocular combinations of luminance increments and decrements. They proposed that separate channels compute the sum and difference of the inputs, and that the information from the two channels is optimally pooled. Quantitatively, their model does not account for many of the quadratic summation phenomena of contrast because their channels are linear.

*Implicit in this qualitative model is a form of half-wave rectification. The model would require that the monocular channels in Fig. 4 give no response to a sine wave shifted 180° from the optimal phase. It would also require spatial-frequency selectivity (presumably a property of the band-pass filter) because the disparity corresponding to a 180° phase shift would depend on the spatial frequency.

In the binocular energy-detector model, all information presented monocularly is funneled through a single, binocular conduit. As a result, it does not include provision for strictly monocular pathways, for which there is some evidence. For example, at low spatial frequencies, subjects can apparently identify the eye of origin of a monocular signal (Blake and Cormack, 1979). Quadratic summation does not preclude the existence of monocular pathways, but does suggest that the variety of phenomena discussed in this paper reflect properties of the binocular pathway.

The value of quadratic summation as a description of binocular contrast summation is two-fold. First, it provides a parameter-free recipe for binocular combination in terms of equivalence relations between monocular and binocular contrast. Second, it is very simple, and gives a reasonable first-order account of a variety of binocular contrast summation phenomena. The value of the binocular energy-detector model is that it accounts for quadratic summation and properties of contrast discrimination within a single theoretical framework.

Acknowledgements—This research was supported by Public Health Service Grant EYO2857. I am particularly grateful to Denis Pelli and Neal Viemeister for helpful discussion of the binocular energy detector. I would also like to thank Gary Rubin for many helpful comments.

REFERENCES

Anderson P. A. and Movshon J. A. (1981) Detection of binocular contrast combinations. *Invest. Ophthal. visual Sci., Suppl.* **20,** 47.

Birch E. E. (1979) Binocular processing of contrast: a vector summation model. *Invest. Ophthal. visual Sci., Suppl.* **18,** 246.

Blake R. and Fox R. (1973) The psychophysical inquiry into binocular summation. *Percept. Psychophys.* **14,** 161–185.

Blake R. and Levinson E. (1977) Spatial properties of binocular neurones in the human visual system. *Expl Brain Res.* **27,** 221–232.

Blake R., Martens W. and Di Gianfilippo A. (1980) Reaction time as a measure of binocular interaction in human vision. *Invest. Ophthal. visual Sci.* **19,** 930–941.

Blake R. and Cormack R. (1979) On utrocular discrimination. *Percept. Psychophys.* **26,** 53–68.

Blakemore C. and Hague B. (1972) Evidence for disparity detecting neurones in the human visual system. *J. Physiol., Lond.* **225,** 437–455.

Burton G. J. (1981) Contrast discrimination by the human visual system. *Biol. Cybernet.* **40,** 27–38.

Campbell F. W. and Green D. G. (1965) Monocular vs. binocular visual acuity. *Nature* **208,** 191–192.

Cannon M. Y. Jr (1979) Contrast sensation: a linear function of stimulus contrast. *Vision Res.* **19,** 1045–1052.

Carlson C. and Cohen R. (1978) A nonlinear spatial frequency signal detection model for the human visual system. *J. opt. Soc. Am.* **68,** 1379.

Cohn T. E. and Lasley D. J. (1976) Binocular vision: two possible central interactions between signals from two eyes. *Science* **192,** 561–563.

Curtis D. W. and Rule S. (1978) Binocular processing of brightness information: a vector sum model. *J. exp Psychol. Hum. Percept. Perform.* **4,** 132–143.

Engel G. R. (1967) The visual processes underlying binocular brightness summation. *Vision Res.* **7,** 753–767.

Engel G. R. (1969) The autocorrelation function and binocular brightness mixing. *Vision Res.,* **9,** 1111–1130.

Felton T. B., Richards W. and Smith R. A. Jr (1972) Disparity processing of spatial frequencies in man. *J. Physiol., Lond.* **225,** 340–362.

Foley J. M. and Legge G. E. (1981) Contrast detection and near-threshold discrimination in human vision. *Vision Res.* **21,** 1041–1053.

Gottesman J., Rubin G. S. and Legge G. E. (1981) A power law for perceived contrast in human vision. *Vision Res.* **21,** 791–799.

Green D. M. and Swets J. A. (1974) *Signal Detection Theory and Psychophysics.* Krieger, Huntington, NY.

Harwerth R. S., Smith E. L. III and Levi D. M. (1980) Suprathreshold binocular interactions for grating patterns. *Percept. Psychophys.* **27,** 43–50.

Iverson G., Movshon J. A. and Arditi A. (1981) Binocular additivity of monocular contrast. *Invest. Ophthal. visual Sci., Suppl.* **20,** 224.

Legge G. E. (1979) Spatial frequency masking in human vision: binocular interactions. *J. opt. Soc. Am.* **69,** 838–847.

Legge G. E. (1981) A power law for contrast discrimination. *Vision Res.* **21,** 457–467.

Legge G. E. (1984) Binocular contrast summation—I. Detection and discrimination. *Vision Res.* **24,** 373–383.

Legge G. E. and Foley J. M. (1980) Contrast masking in human vision. *J. opt. Soc. Am.* **70,** 1458–1471.

Legge G. E. and Rubin G. S. (1981) Binocular interactions in suprathreshold contrast perception. *Percept. Psychophys.* **30,** 49–61.

Legge G. E. and Kersten D. (1983) Light and dark bars: contrast discrimination. *Vision Res.* **23,** 473–483.

Levelt W. J. M. (1965) *On Binocular Rivalry.* R.V.O.T.N.O., Soesterberg, The Netherlands.

Levi D. M., Harwerth R. S. and Smith E. L. III (1979) Humans deprived of normal binocular vision have binocular interactions tuned to size and orientation. *Science* **206,** 852–854.

Linfoot E. H. (1964) *Fourier Methods in Optical Image Evaluation.* The Focal Press, London.

Pelli, D. G. (1981) *The Effects of Visual Noise.* Ph.D. Thesis, Cambridge Univ.

Rashbass C. (1970) Visibility of transient changes of luminance. *J. Physiol., Lond.* **210,** 165–186

Rubin G. S. (1983) *Suppression and Summation in Binocular Pattern Vision.* Ph.D. Thesis, Univ. of Minnesota.

Stevens J. C. (1967) Brightness function: binocular vs monocular stimulation. *Percept. Psychophys.* **2,** 451–454.

Tolhurst D. J., Movshon J. A. and Thompson I. D. (1981) The dependence of response amplitude and variance of cat visual cortical neurones on stimulus contrast. *Expl Brain Res.* **41,** 414–419.

Wetherill G. B. and Levitt H. (1965) Sequential estimation of points on a psychometric function. *Br. J. Math. Stat. Psychol.* **18,** 1–10.

Wilson H. R. (1980) A transducer function for threshold and suprathreshold spatial vision. *Biol. Cybernet.* **38,** 171–178.